

## AMENDMENTS TO THE CLAIMS

### Claims Pending:

- At time of the Action: Claims 1-63
- Amended Claims: Claims 1, 2, 5, 6, 9, 10, 13, 14, 16, 22, 30-48, 50-54, 56, 58-63
- Allowable Claims: Claims 4, 8, and 12
- Cancelled Claims: Claims 3, 4, 7, 8, 11, and 12
- After this Response: Claims 1, 2, 5, 6, 9, 10, and 13-63

This listing of claims will replace all prior versions and listings, of claims in the application.

1. (Currently Amended) A method comprising:  
determining at least one Squared Tate pairing for at least one hyperelliptic curve; and  
wherein determining the Squared Tate pairing further includes:  
forming a mathematical chain for m, wherein m is a positive integer and an m-torsion  
element D is fixed on Jacobian of the hyperelliptic curve C;  
wherein the mathematical chain includes a mathematical chain selected from a group  
of mathematical chains comprising an addition chain and an addition-subtraction chain;  
cryptographically processing selected information based on said the determined  
Squared Tate pairing;  
outputting validation of selected information based on the determined Squared Tate  
pairing; and  
determining a course of action in response to validation of selected information.

2. (Currently Amended) The method as recited in Claim 1, wherein said the Squared Tate pairing is defined for at least one hyperelliptic curve  $C$  of genus  $g$  over a field  $K$ .

3.-4. (Cancelled).

5. (Currently Amended) A computer-readable storage medium having computer-implementable instructions for causing at least one processing unit to perform acts comprising:

calculating at least one Squared Tate pairing for at least one hyperelliptic curve; and  
wherein determining the Squared Tate pairing further includes:  
forming a mathematical chain for  $m$ , wherein  $m$  is a positive integer and an  $m$ -torsion element  $D$  is fixed on Jacobian of the hyperelliptic curve  $C$ :

wherein the mathematical chain includes a mathematical chain selected from a group of mathematical chains comprising an addition chain and an addition-subtraction chain;

cryptographically processing selected information based on said the determined Squared Tate pairing;

outputting validation of selected information based on the determined Squared Tate pairing; and

determining a course of action in response to validation of selected information.

6. (Currently Amended) The computer-readable storage medium as recited in Claim 5, wherein said the Squared Tate pairing is defined for at least one hyperelliptic curve  $C$  of genus  $g$  over a field  $K$ .

7.-8. (Cancelled).

9. (Currently Amended) An apparatus comprising:  
memory configured to store information suitable for use with using a cryptographic process;  
logic operatively coupled to said the memory and configured to calculate at least one Squared Tate pairing for at least one hyperelliptic curve, and at least partially support cryptographic processing of selected stored information based on said the determined Squared Tate pairing;

wherein the logic is further configured to form a mathematical chain for  $m$ , wherein  $m$  is a positive integer and an  $m$ -torsion element  $D$  is fixed on Jacobian of the hyperelliptic curve  $C$ ;

wherein the mathematical chain includes a mathematical chain selected from a group of mathematical chains comprising an addition chain and an addition-subtraction chain;

a display device coupled to the logic for outputting validation of selected information; and

the logic determining a course of action in response to validation.

10. (Currently Amended) The apparatus as recited in Claim 9, wherin said the Squared Tate pairing is defined for at least one hyperelliptic curve  $C$  of genus  $g$  over a field  $K$ .

11.-12. (Cancelled).

13. (Currently Amended) A method comprising:  
determining a hyperelliptic curve  $C$  of genus  $g$  over a field  $K$  and a positive integer  $m$ ;  
determining a Jacobian  $J(C)$  of said the hyperelliptic curve  $C$ , and wherin each element  $D$  of  $J(C)$  contains a representative of the form  $A - g(P_0)$ , where  $A$  is an effective divisor of degree  $g$ ; and  
determining a plurality of functions  $h_{j,D}$  that are iterative building blocks for the formation of a function  $h_{m,D}$  in order to evaluate  $v_m$  which is a Squared Tate pairing;  
outputting validation of selected information based on the Squared Tate pairing; and  
determining a course of action in response to validation of selected information.

14. (Currently Amended) The method as recited in Claim 13, wherin said the hyperelliptic curve  $C$  is over a field not of characteristic 2.

15. (Original) The method as recited in Claim 13, wherin  
for at least one element  $D$  of  $J(C)$ , a representative for  $iD$  will be  $A_i - g(P_0)$ , where  $A_i$  is effective of degree  $g$ .

16. (Currently amended) The method as recited in Claim 13, wherein if  $P=(x, y)$  is a point on said the hyperelliptic curve  $C$ , then  $-P$  denotes a point  $-P:=(x, -y)$ , and wherein if a point  $P=(x, y)$  occurs in  $A$  and  $y \neq 0$ , then  $-P:=(x, -y)$  does not occur in  $A$  and a representative for identity will be  $g(P_0)$ .

17. (Original) The method as recited in Claim 16, further comprising:  
to a representative  $A_t$ , associating two polynomials  $(a_t, b_t)$  which represent a divisor.

18. (Original) The method as recited in Claim 16, further comprising:  
determining  $D$  as an  $m$ -torsion element of  $J(C)$ .

19. (Original) The method as recited in Claim 18, further comprising:  
if  $j$  is an integer, then  $h_{j,D} = h_{j,D}(X)$  denoting a rational function on  $C$  with divisor  $(h_{j,D}) = jA_1 - A_j - ((j-1)g)(P_0)$ .

20. (Original) The method as recited in Claim 18, wherein  $D$  is an  $m$ -torsion divisor and  $A_m = g(P_0)$ , and a divisor of  $h_{m,D}$  is  $(h_{m,D}) = mA_1 - mg(P_0)$ .

21. (Original) The method as recited in Claim 18, wherein  $h_{m,D}$  is well-defined up to a multiplicative constant.

22. (Currently Amended) The method as recited in Claim 18, further comprising:  
evaluating  $h_{m,D}$  at a degree zero divisor  $E$  on said the hyperelliptic curve  $C$ , wherein  
 $E$  does not contain  $P_0$  and  $E$  is prime to  $A_i$ .

23. (Original) The method as recited in Claim 18, wherein  $E$  is prime to  $A_i$  for all  
 $i$  in an addition-subtraction chain for  $m$ .

24. (Original) The method as recited in Claim 22, wherein given  $A_i$ ,  $A_j$ , and  $A_{i+j}$ ,  
further comprising determining a function  $u_{i,j}$  such that a divisor of  $u_{i,j}$  is  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ .

25. (Original) The method as recited in Claim 22, further comprising:  
evaluating  $h_{j,D}(E)$  such that when  $j=1$ ,  $h_{1,D}$  is 1.

26. (Original) The method as recited in Claim 22, further comprising:  
given  $A_i$ ,  $A_j$ ,  $h_{i,D}(E)$  and  $h_{j,D}(E)$ , evaluating  $u_{i,j}$  to be  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ , and  
 $h_{i+j,D}(E) = h_{i,D}(E) h_{j,D}(E) u_{i,j}(E)$ .

27. (Original) The method as recited in Claim 13, further comprising:  
determining a function  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ .

28. (Original) The method as recited in Claim 27, wherein  $g = 2$  and  
 $(u_{i,j}) = A_i + A_j - A_{i+j} - 2(P_0)$  is determined as follows

$$u_{i,j}(\mathbf{X}) := \frac{a_{\text{new}}(x(\mathbf{X}))}{b_{\text{new}}(x(\mathbf{X})) + y(\mathbf{X})} * d(x(X)), \text{ if the degree of } a_{\text{new}} \text{ is}$$

greater than 2, otherwise,  $u_{i,j}$  is determined as  $u_{i,j}(X) := d(x(X))$ , wherein  $d(x)$  is the greatest common divisor of three polynomials  $(a_i(x), a_j(x), b_i(x)+b_j(x))$ .

29. (Original) The method as recited in Claim 13, further comprising:  
determining a Squared Tate pairing for a hyperelliptic curves  $v_m$ , for an  $m$ -torsion element  $D$  of a Jacobian  $J(C)$  and an element  $E$  of  $J(C)$ , with representatives  $(P_1)+(P_2)+\dots+(P_g)-g(P_0)$  and  $(Q_1)+(Q_2)+\dots+(Q_g)-g(P_0)$ , respectively, with each  $P_i$  and each  $Q_j$  on the curve  $C$ , with  $P_i$  not equal to  $\pm Q_j$  for all  $i, j$ , determining that

$$v_m(D, E) := (h_{m,D}((\mathbf{Q}_1) - (-\mathbf{Q}_1) + (\mathbf{Q}_2) - (-\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) - (-\mathbf{Q}_g))^{\frac{q-1}{m}}.$$

30. (Currently Amended) A computer-readable storage medium having computer-implementable instructions for causing at least one processing unit to perform acts comprising:

determining a hyperelliptic curve  $C$  of genus  $g$  over a field  $K$  and a positive integer  $m$ ;

determining a Jacobian  $J(C)$  of said the hyperelliptic curve  $C$ , and wherein each element  $D$  of  $J(C)$  contains a representative of the form  $A - g(P_0)$ , where  $A$  is an effective divisor of degree  $g$ ; and

determining a plurality of functions  $h_{j,D}$  that are iterative building blocks for the formation of a function  $h_{m,D}$  in order to evaluate  $v_m$  which is a Squared Tate pairing;

outputting validation of selected information based on the Squared Tate pairing; and

determining a course of action in response to validation of selected information.

31. (Currently Amended) The computer-readable storage medium as recited in Claim 30, wherein said the hyperelliptic curve  $C$  is not of characteristic 2.

32. (Currently Amended) The computer-readable storage medium as recited in Claim 30, wherein

for at least one element  $D$  of  $\mathcal{J}(C)$ , a representative for  $iD$  will be  $A_i - g(P_0)$ , where  $A_i$  is effective of degree  $g$ .

33. (Currently Amended) The computer-readable storage medium as recited in Claim 30, wherein if  $P=(x, y)$  is a point on said the hyperelliptic curve  $C$ , then  $-P$  denotes a point  $-P:=(x, -y)$ , and wherein if a point  $P=(x, y)$  occurs in  $A$  and  $y \neq 0$ , then  $-P:=(x, -y)$  does not occur in  $A$  and a representative for identity will be  $g(P_0)$ .

34. (Currently Amended) The computer-readable storage medium as recited in Claim 33, further comprising:

to a representative  $A_i$ , associating two polynomials  $(a_i, b_i)$  which represent a divisor.

35. (Currently Amended) The computer-readable storage medium as recited in Claim 33, further comprising:

determining  $D$  as an  $m$ -torsion element of  $\mathcal{J}(C)$ .

36. (Currently Amended) The computer-readable storage medium as recited in Claim 35, further comprising:

if  $j$  is an integer, then  $h_{j,D} = h_{j,D}(X)$  denoting a rational function on  $C$  with divisor  $(h_{j,D}) = jA_1 - A_j - ((j-1)g)(P_0)$ .

37. (Currently Amended) The computer-readable storage medium as recited in Claim 35, wherein  $D$  is an  $m$ -torsion divisor and  $A_m = g(P_0)$ , and a divisor of  $h_{m,D}$  is  $(h_{m,D}) = mA_1 - mg(P_0)$ .

38 (Currently Amended) The computer-readable storage medium as recited in Claim 35, wherein  $h_{m,D}$  is well-defined up to a multiplicative constant.

39. (Currently Amended) The computer-readable storage medium as recited in Claim 35, further comprising:

evaluating  $h_{m,D}$  at a degree zero divisor  $E$  on said the hyperelliptic curve  $C$ , wherein  $E$  does not contain  $P_0$  and  $E$  is prime to  $A_i$ .

40. (Currently Amended) The computer-readable storage medium as recited in Claim 35, wherein  $E$  is prime to  $A_i$  for all  $i$  in an addition-subtraction chain for  $m$ .

41. (Currently Amended) The computer-readable storage medium as recited in Claim 39, wherein given  $A_i$ ,  $A_j$ , and  $A_{i+j}$ , further comprising determining a function  $u_{i,j}$  such that a divisor of  $u_{i,j}$  is  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ .

42. (Currently Amended) The computer-readable storage medium as recited in Claim 39, further comprising:

evaluating  $h_{j,D}(E)$  such that when  $j=1$ ,  $h_{1,D}$  is 1.

43. (Currently Amended) The computer-readable storage medium as recited in Claim 39, further comprising:

given  $A_i$ ,  $A_j$ ,  $h_{i,D}(E)$  and  $h_{j,D}(E)$ , evaluating  $u_{i,j}$  to be  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ , and  $h_{i+j,D}(E) = h_{i,D}(E) \cdot h_{j,D}(E) \cdot u_{i,j}(E)$ .

44. (Currently Amended) The computer-readable storage medium as recited in Claim 30, further comprising:

determining a function  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ .

45. (Currently Amended) The computer-readable storage medium as recited in Claim 44, wherein  $g = 2$  and

$(u_{i,j}) = A_i + A_j - A_{i+j} - 2(P_0)$  is determined as follows

$$u_{i,j}(\mathbf{X}) = \frac{a_{\text{new}}(x(\mathbf{X}))}{b_{\text{new}}(x(\mathbf{X})) + y(\mathbf{X})} * d(x(\mathbf{X})), \text{ if the degree of } a_{\text{new}} \text{ is}$$

greater than 2, otherwise,  $u_{i,j}$  is determined as  $u_{i,j}(X) := d(x(X))$ , wherein  $d(x)$  is the greatest common divisor of three polynomials  $(a_i(x), a_j(x), b_i(x)+b_j(x))$ .

46. (Currently Amended) The computer-readable storage medium as recited in Claim 30, further comprising:

determining a Squared Tate pairing for a hyperelliptic curves  $v_m$ , for an  $m$ -torsion element  $D$  of a Jacobian  $J(C)$  and an element  $E$  of  $J(C)$ , with representatives  $(P_1)+(P_2)+\dots+(P_g) - g(P_0)$  and  $(Q_1)+(Q_2)+\dots+(Q_g) - g(Q_0)$ , respectively, with each  $P_i$  and each  $Q_j$  on the curve  $C$ , with  $P_i$  not equal to  $\pm Q_j$  for all  $i, j$ , determining that

$$v_m(D, E) := (h_{m,D}((Q_1) - (-Q_1) + (Q_2) - (-Q_2) + \dots + (Q_g) - (-Q_g))^{\frac{g-1}{m}}.$$

47. (Currently Amended) An apparatus comprising:  
memory configured to store information suitable for use with using a cryptographic process; and

logic operatively coupled to said the memory and configured to determine a hyperelliptic curve  $C$  of genus  $g$  over a field  $K$  and a positive integer  $m$ , determine a Jacobian  $J(C)$  of said the hyperelliptic curve  $C$ , wherein each element  $D$  of  $J(C)$  contains a representative of the form  $A - g(P_0)$  and  $A$  is an effective divisor of degree  $g$ , and determine a plurality of functions  $h_{j,D}$  that are iterative building blocks for the formation of a function  $h_{m,D}$  in order to evaluate  $v_m$  which is a Squared Tate pairing;

a display device coupled to the logic for outputting validation of selected information;  
and

the logic determining a course of action in response to the validation.

48. (Currently Amended) The apparatus as recited in Claim 47, wherein said the hyperelliptic curve  $C$  is not of characteristic 2.

49. (Original) The apparatus as recited in Claim 47, wherein  
for at least one element  $D$  of  $J(C)$ , a representative for  $iD$  will be  $A_i - g(P_0)$ , where  $A_i$  is effective of degree  $g$ .

50. (Currently Amended) The apparatus as recited in Claim 47, wherein if  $P=(x, y)$  is a point on the hyperelliptic curve  $C$ , then  $-P$  denotes a point  $-P:=(x, -y)$ , and wherein if a point  $P=(x, y)$  occurs in  $A$  and  $y \neq 0$ , then  $-P := (x, -y)$  does not occur in  $A$  and a representative for identity will be  $g(P_0)$ .

51. (Currently Amended) The apparatus as recited in Claim 50, wherein said the logic is further configured to, for a representative  $A_i$ , associate two polynomials  $(a_i, b_i)$  which represent a divisor.

52. (Currently Amended) The apparatus as recited in Claim 50, wherein said the logic is further configured to determine  $D$  as an  $m$ -torsion element of  $J(C)$ .

53. (Currently Amended) The apparatus as recited in Claim 52, wherein said the logic is further configured to, if  $j$  is an integer, then determine  $h_{j,D} = h_{j,D}(X)$  by denoting a rational function on  $C$  with divisor  $(h_{j,D}) = jA_1 - A_j - ((j-1)g)(P_0)$ .

54. (Currently Amended) The ~~computer-readable medium apparatus~~ as recited in Claim 52, wherein  $D$  is an  $m$ -torsion divisor and  $A_m = g(P_0)$ , and a divisor of  $h_{m,D}$  is  $(h_{m,D}) = mA_1 - mg(P_0)$ .

55 (Original) The apparatus as recited in Claim 52, wherein  $h_{m,D}$  is well-defined up to a multiplicative constant.

56. (Currently Amended) The apparatus as recited in Claim 52, wherein said the logic is further configured to evaluate  $h_{m,D}$  at a degree zero divisor  $E$  on said the hyperelliptic curve  $C$ , wherein  $E$  does not contain  $P_0$  and  $E$  is prime to  $A_i$ .

57. (Original) The apparatus as recited in Claim 52, wherein  $E$  is prime to  $A_i$  for all  $i$  in an addition-subtraction chain for  $m$ .

58. (Currently Amended) The apparatus as recited in Claim 56, wherein given  $A_i$ ,  $A_j$ , and  $A_{i+j}$ , and wherein said the logic is further configured to determine a function  $u_{i,j}$  such that a divisor of  $u_{i,j}$  is  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ .

59. (Currently Amended) The apparatus as recited in Claim 56, wherein said the logic is further configured to evaluate  $h_{j,D}(E)$  such that when  $j=1$ ,  $h_{1,D}$  is 1.

60. (Currently Amended) The apparatus as recited in Claim 56, wherein said the logic is further configured to, given  $A_i$ ,  $A_j$ ,  $h_{i,D}(E)$  and  $h_{j,D}(E)$ , evaluate  $u_{i,j}$  to be  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ , and  $h_{i+j,D}(E) = h_{i,D}(E) \ h_{j,D}(E) \ u_{i,j}(E)$ .

61. (Currently Amended) The apparatus as recited in Claim 47, wherein said the logic is further configured to determine a function  $(u_{i,j}) = A_i + A_j - A_{i+j} - g(P_0)$ .

62. (Currently Amended) The apparatus as recited in Claim 61, wherein  $g = 2$  and

$(u_{i,j}) = A_i + A_j - A_{i+j} - 2(P_0)$  is determined by said the logic as follows

$$u_{i,j}(\mathbf{X}) := \frac{a_{\text{new}}(x(\mathbf{X}))}{b_{\text{new}}(x(\mathbf{X})) + y(\mathbf{X})} * d(x(X)), \text{ if the degree of } a_{\text{new}} \text{ is}$$

greater than 2, otherwise,  $u_{i,j}$  is determined as  $u_{i,j}(X) := d(x(X))$ , wherein  $d(x)$  is the greatest common divisor of three polynomials  $(a_i(x), a_j(x), b_i(x) + b_j(x))$ .

63. (Currently Amended) The apparatus as recited in Claim 47, wherein said the logic is further configured to determine a Squared Tate pairing for a hyperelliptic curves  $v_m$ , for an  $m$ -torsion element  $D$  of a Jacobian  $J(C)$  and an element  $E$  of  $J(C)$ , with representatives  $(P_1) + (P_2) + \dots + (P_g) - g(P_0)$  and  $(Q_1) + (Q_2) + \dots + (Q_g) - g(P_0)$ , respectively, with each  $P_i$  and each  $Q_j$  on the curve  $C$ , with  $P_i$  not equal to  $\pm Q_j$  for all  $i, j$ , and to determine that

$$v_m(D, E) := (h_{m,D}((\mathbf{Q}_1) - (-\mathbf{Q}_1) + (\mathbf{Q}_2) - (-\mathbf{Q}_2) + \dots + (\mathbf{Q}_g) - (-\mathbf{Q}_g)))^{\frac{q-1}{m}}.$$